NAVAL POSTGRADUATE SCHOOL
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THESIS

AN ALGEBRAIC STRUCTURE FOR THE
CONVOLUTION OF LIFE DISTRIBUTIONS

by

Danny L. Hogg

October 1982

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An Algebraic Structure for the Convolution of Life Distributions

by

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In this paper one method for analytically describing the life distribution of a system is investigated. This is done by using the inherent properties of convolutions and mixtures of life distributions to create an algebraic structure. Once the algebraic structure is constructed it can be used to develop algorithms to go from the schematic of a system to its survival function. It is noted along the way that many combinations of constant failure rate components, e.g., redundant, series, or parallel systems can be described by a mixture of convolutions and that often these expressions can be greatly simplified.

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An algebraic structure has been derived for the combination of life distributions which describe the reliability of systems. Such a structure consists of a set of elements $D_1, D_2, \ldots, D_n$ combined by operations such as addition or multiplication. The set of elements in the algebraic structure derived here is life distributions. Each life distribution is assumed to have a probability density function. An exception is the ZERO distribution defined later. The random variable associated with the distribution is time. Since a life distribution can be fully described by its survival function, that representation will often be used. The operations used in this algebraic structure will not be addition and multiplication but the operations of $\oplus$ and MIX. They are defined as follows:

$D_1 \oplus D_2$ is the convolution of two life distributions. Using survival functions:

$$D_1 \oplus D_2 \Rightarrow \overline{F}_1(t) + \int_0^t \overline{F}_2(t-s) f_1(s) ds$$

In words this says the probability that the system resulting from $D_1 \oplus D_2$ will survive till time t, is the probability that the component whose life distribution is D_1 will survive till time t plus the probability that the component whose life distribution is D_2 will survive from time s until t given

that component one (whose life distribution is D_1) lived until time s and failed at that moment. It will be shown later that the probability the system created by the convolution will survive till time t is the same regardless of whether D_1 or D_2 is considered first.

MIX is the mixing or combination of life distributions with a priori mixing probabilities. In terms of survival functions:

$$MIX[p_1D_1, p_2D_2] \Rightarrow p_1\bar{F}_1(t) + p_2\bar{F}_2(t).$$

This relation says the probability that the system resulting from $MIX[p_1D_1, p_2D_2]$ will survive till time t is the sum of the probabilities that either component will survive till time t multiplied by their a priori mixing probabilities. These operations display certain algebraic properties such as commutativity, associativity, distributivity, and the presence of an identity. The following theorem summarizes these properties. The proof for the theorem is manipulative. The theorem is true more generally than for just independent and continuous distributions for nonnegative random variables, but only this case is shown here.

THEOREM: THE SET OF LIFE DISTRIBUTIONS $D_1, D_2, \ldots, D_n$ FORM A MONOID UNDER THE OPERATIONS OF $\oplus$ AND MIX.

PROOF:

(1) Commutativity

(a) $D_1 \oplus D_2 \Rightarrow \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s) ds$

The fact that $F(t) = 1 - \bar{F}(t)$ yields:

$$1 - \int_0^t f_1(s) F_2(t-s) ds.$$

Using the convolution property of integrals yields:

$$1 - \int_0^t f_1(t-s) F_2(s) ds.$$

Now applying integration by parts we have:

$$1 - \int_0^t F_1(t-s) f_2(s) ds.$$

And if the previous steps are retraced:

$$D_1 \oplus D_2 \Rightarrow \bar{F}_2(t) + \int_0^t \bar{F}_1(t-s) f_2(d) ds$$

which is the survival function for the life distribution $D_2 \oplus D_1$.

(b) $MIX[p_1D_1,p_2D_2] \Rightarrow p_1\overline{F}_1(t) + p_2\overline{F}_2(t)$.

Applying the commutative property of normal addition yields $p_2\overline{F}_2(t) + p_1\overline{F}_1(t)$ which is the survival function for $MIX[p_2D_2,p_1D_1]$.

(2) Associativity

(a) $(D_1 \oplus D_2) \oplus D_3 \Rightarrow \overline{F}_{1+2}(t) + \int_0^t \overline{F}_3(t-s)f_{1+2}(s)ds$

where $\overline{F}_{1+2}(t) = \overline{F}_1(t) + \int_0^t \overline{F}_2(t-s)f_1(s)ds$

and $f_{1+2}(s) = \int_0^s f_2(s-u)f_1(u)du$.

Making the substitutions and invoking the associative property of convolution integrals yields:

$$\overline{F}_1(t) + \int_0^t \overline{F}_2(t-s)f_1(s)ds + \int_0^t \int_0^{t-s} \overline{F}_3(t-s-u)f_1(s) \times f_2(u)duds.$$

This integral equation reduces to:

$$\overline{F}_1(t) + \int_0^t \overline{F}_{2+3}(t-s)f_1(s)ds,$$

which is the survival function for the life distribution $D_1 \oplus (D_2 \oplus D_3)$.

(b) $\text{MIX}[(p_1D_1,p_2D_2),p_3D_3] \Rightarrow$

$$(p_1\overline{F}_1(t) + p_2\overline{F}_2(t)) + p_3\overline{F}_3(t).$$

Applying the associative property of normal addition yields:

$$p_1\overline{F}_1(t) + (p_2\overline{F}_2(t) + p_3\overline{F}_3(t))$$

which is the survival function for

$$\text{MIX}[p_1D_1,(p_2D_2,p_3D_3)].$$

(3) Distributivity

$D_1 \oplus \text{MIX}[p_2D_2,p_3D_3] \Rightarrow$

$$\overline{F}_1(t) + \int_0^t (p_2\overline{F}_2(t-s) + p_3\overline{F}_3(t-s))f_1(s)ds$$

Letting $p_2 + p_3 = 1$ results in:

$$p_2\overline{F}_1(t) + p_3\overline{F}_1(t) + \int_0^t p_2\overline{F}_2(t-s)f_1(s)ds$$

$$+ \int_0^t p_3\overline{F}_3(t-s)f_1(s)ds$$

which implies $\text{MIX}[p_2(D_1 \oplus D_2),p_3(D_1 \oplus D_3)]$.

It is assumed in the MIX operation that the assigned a priori probabilities will sum to one. If the sum of the a

priori probabilities is less than one, then the life distribution resulting from the MIX operation will be an improper distribution, but this can be remedied by the use of the ZERO distribution. ZERO will act as an identity element for both $\oplus$ and MIX. By definition $\overline{\text{ZERO}}(t) = 0$; this means the probability of a component (with a ZERO life distribution) being alive at time $t > 0$ is 0.

(4) Identity

(a) i. Right identity

$$D_1 \oplus \text{ZERO} \Rightarrow \overline{F}_1(t) + \int_0^t \overline{\text{ZERO}}(t-s) f_1(s)ds$$

This equals $\overline{F}_1(t)$ which is the survival function for just D_1.

ii. Left identity

There is not a comparable form for showing ZERO to be a left identity, but by employing an extended commutative property first the needed result can be obtained.

(b) $\text{MIX}[p_1D_1, p_2\text{ZERO}] \Rightarrow p_1\overline{F}_1(t) + p_2\overline{\text{ZERO}}(t)$

This equals $p_1\overline{F}_1(t)$ which implies p_1D_1. The left identity for MIX is obvious using the commutative property.

This argument establishes all the properties necessary for the operations of $\oplus$ and MIX to form a monoid over the set of continuous life distributions with ZERO adjoined. The next question would be to ask if this set of operations form a group. The answer is negative since there does not exist a unique inverse for each element in the set of life distributions.

There is another property that could prove valuable in the manipulation of life distributions. That property is the idempotence property for the MIX operation, i.e.,

$$MIX[p_1 D, p_2 D] \Rightarrow D$$

It is a further observation that a mixture of mixtures is a mixture.

II. LIFE DISTRIBUTIONS AND BRANCHING DIAGRAMS

The power of the previous algebraic properties can be most easily seen using exponential life distributions. As an example, the life distribution of a redundant system with failure rates of the primary and backup components of λ_1 and λ_2 respectively, can be described as $\mathrm{EXP}\{\lambda_1\} \oplus \mathrm{EXP}\{\lambda_2\}$. $\mathrm{EXP}\{\lambda\}$ is a convenient shorthand to describe an exponential life distribution with failure rate λ. The survival function for the system will be the convolution of the survival functions for the two components. It is given by

$$e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s} ds$$

which simplifies to:

$$\frac{\lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + \frac{\lambda_2 e^{-\lambda_2 t}}{\lambda_1 - \lambda_2}$$

This equation is symmetric in λ_1 and λ_2, hence the operation is commutative. The form of this solution can be extended to the nth case $\mathrm{EXP}\{\lambda_1\} \oplus \mathrm{EXP}\{\lambda_2\} \oplus \ldots \oplus \mathrm{EXP}\{\lambda_n\}$. The survival function is:

$$\sum_{i=1}^{n} \frac{\prod_{i \neq j} \lambda_j e^{-\lambda_i t}}{\prod_{i \neq j} (\lambda_j - \lambda_i)}$$

In the case $\lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_n$ the closed form of the solution is:

$$\sum_{k=1}^{n} \frac{(\lambda t)^{k-1}}{(k-1)!} e^{-\lambda t}$$

If two exponentially lived components are connected in series, the life distribution of the system will be $EXP\{\lambda_1 + \lambda_2\}$, and the survival function is:

$$e^{-(\lambda_1+\lambda_2)t}$$

If two components are connected in parallel the life distribution of the system is:

$$EXP\{\lambda_1+\lambda_2\} \oplus MIX[\frac{\lambda_1}{\lambda_1+\lambda_2} EXP\{\lambda_2\}, \frac{\lambda_2}{\lambda_1+\lambda_2} EXP\{\lambda_1\}],$$

where the *a priori* mixing probabilities are the probabilities that $EXP\{\lambda_1\}$ will fail before $EXP\{\lambda_2\}$, and $EXP\{\lambda_2\}$ will fail before $EXP\{\lambda_1\}$, respectively. The survival function for the parallel system is:

$$e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

Both the series and parallel system can be extended to the case of n components.

These various simple systems can be connected to form more complex systems, but all can be analytically described by a mixture of convolutions. It will be necessary to adopt a convention to describe graphically the life distribution of these complex systems. This convention will be a branching diagram as seen in Figure 2.1.

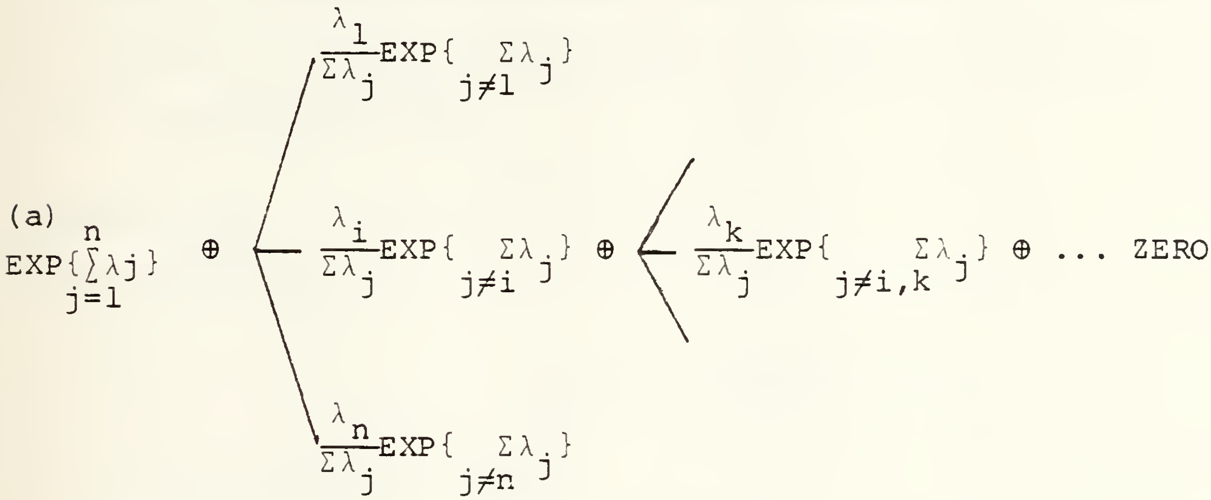


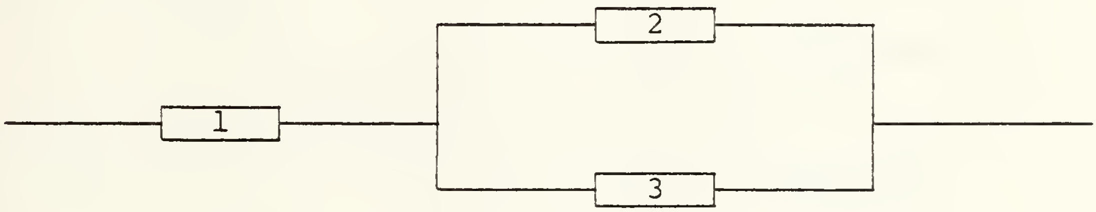
Figure 2.1. Branching Diagram

At the base of the diagram will be (a), the probability distribution for the survival of all the components. Branching from the base are different paths that represent the mixture of the life distributions of the remaining components given that one or more of the original components has failed. The life distribution at point (a) will be convolved with the mixture of the branches. Each of these branches is given an a priori probability of occurrence. Each of these paths

may again branch, giving rise to a new mixture which will be convolved with preceding life distribution. These branches will continue until the failure of any component will result in the failure of the complete system. When all the branches have been drawn out to completion, the sum of the products of the a priori probabilities along each branch must be one. It may be necessary to use the ZERO distribution, as described in Chapter I, to achieve this sum. A couple of examples will clarify the point.

Example 2a)

The schematic:



The convolution diagram:

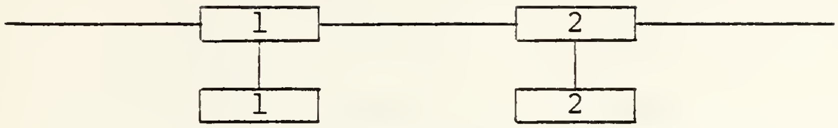
$$\mathrm{EXP}\{\lambda_1+\lambda_2+\lambda_3\} \oplus \left< \begin{array}{l} \dfrac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\mathrm{ZERO} \\[2ex] \dfrac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\mathrm{EXP}\{\lambda_1+\lambda_3\} \\[2ex] \dfrac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\mathrm{EXP}\{\lambda_1+\lambda_2\} \end{array} \right.$$

The life distribution:

$$\mathrm{EXP}\{\lambda_1+\lambda_2+\lambda_3\} \oplus \mathrm{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\mathrm{ZERO}, \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\mathrm{EXP}\{\lambda_1+\lambda_3\}, \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\mathrm{EXP}\{\lambda_1+\lambda_2\}\right]$$

Example 2b)

The schematic:



The convolution diagram:

$$\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \begin{cases} \dfrac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \begin{cases} \dfrac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{ZERO} \\ \dfrac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\} \end{cases} \\ \\ \dfrac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \begin{cases} \dfrac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\} \\ \dfrac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{ZERO} \end{cases} \end{cases}$$

The life distribution:

$$\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \mathrm{MIX}[\frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \mathrm{MIX}[\frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{ZERO}, \frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\}],$$

$$\frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \mathrm{MIX}[\frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1+\lambda_2\}, \frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{ZERO}]]$$

Writing out these complete life distributions can be quite long and tedious, but by invoking the distributive and idempotent properties shown earlier, we can move each life

distribution from outside the MIX brackets to the inside, and express MIX of a MIX as a single MIX. The life distribution in the second example would become:

$$\mathrm{MIX}\left[\frac{\lambda_1^2 + \lambda_2^2}{(\lambda_1+\lambda_2)^2}(\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \mathrm{EXP}\{\lambda_1+\lambda_2\}),\right.$$

$$\left.\frac{2\lambda_1\lambda_2}{(\lambda_1+\lambda_2)^2}(\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \mathrm{EXP}\{\lambda_1+\lambda_2\})\right]$$

By using the above algorithm to express the life distribution of a complex system, we have proven the following theorem:

THEOREM: THE LIFE DISTRIBUTION OF ANY SYSTEM WHICH CAN BE REPRESENTED BY A BRANCHING DIAGRAM WITH EXPONENTIAL LIFE DISTRIBUTIONS ALONG THE BRANCHES CAN BE EXPRESSED AS A MIXTURE OF CONVOLUTIONS OF EXPONENTIAL LIFE DISTRIBUTIONS.

By further applying the algebraic properties some very simple expressions can be derived from some very complex ones. A good example of one such identity is:

$$\mathrm{EXP}\{\lambda_1+\lambda_2\} \oplus \mathrm{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{ZERO},\frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}\{\lambda_1\}\right] \Rightarrow \mathrm{EXP}\{\lambda_1\}$$

III. THE CORRESPONDENCE BETWEEN BRANCHING DIAGRAMS AND FAULT TREES

In the preceeding chapter the notion of a branching diagram was introduced. We can also graphically display complex systems and the effect of the failure of some components on the whole system by the method of fault trees. The class of systems which can be represented by fault trees and the class of systems which can be represented by branching diagrams are not identical. There exist systems which can be represented by branching diagrams which cannot be represented by fault trees. An example of such consists of systems involving standby redundancy. In this chapter it will be shown that, given a system that can be represented by a fault tree, the length of the paths of the branching diagram can be determined by the minimum cut sets from the fault tree. It is assumed that the components fail independently of each other and the life distribution of those components are exponential.

The fault tree provides a convenient and efficient format helpful in the computation of the probability of system success or failure. The fault tree consists of boxes representing basic events, AND gates, and OR gates. The top box in the tree will represent system failure. If the top box had represented system success the fault tree would have become an event tree. Immediately below the top event will

be a gate with lines leading to the next level of events. If the gate is an AND gate, marked with a ·, then all the events on the next level must occur to cause the top event to occur. If the gate is an OR gate, marked by a +, then the occurrence of any of the events of the next level will cause the top event to occur. The tree will continue to grow until all possible events have been considered. Once the tree has been drawn it is an easy chore to write down the minimum cuts. A minimum cut is defined to be a minimum set necessary for the top event to occur. The algorithm to find such cuts is taken from Barlow and Proschan's Statistical Theory of Reliability and Life Testing [Ref. 1: p. 256]. The algorithm begins with the gate immediately below the top event. If the gate is an OR gate, each input is used as an entry in separate rows of a list matrix. If this gate

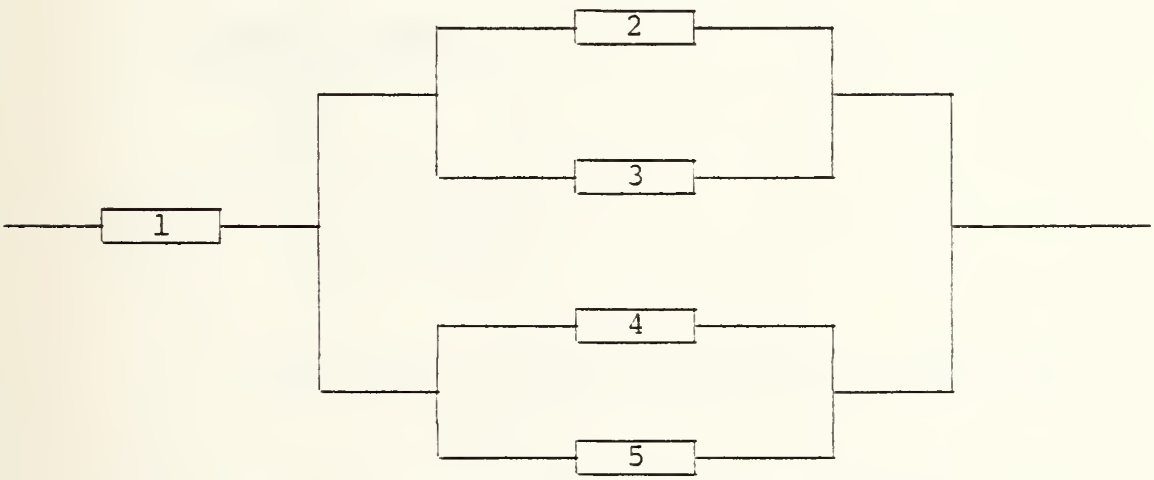
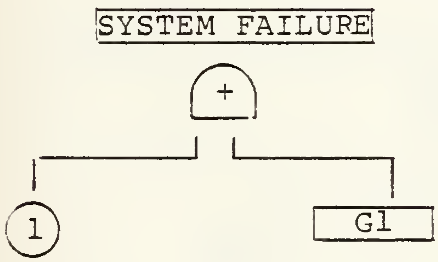


Figure 3.1. Schematic for Example 3a

is an AND gate, each input is used as an entry in the first row of a list matrix. If one of these inputs is another gate, then the inputs to that gate are listed in the same or separate rows of the list matrix according to the nature of the gate. Multiple entries in the rows of the list matrix are the result of AND gates. A row with entry a,b implies that this minimum cut will occur if both components a and b fail. Once the list contains all components and no gates then the minimum cuts can be read across each row. Example 3a is given to clarify this point.

Example 3a) Given the schematic in Figure 3.1 it is obvious that the system will fail if either component one or all the other components in the parallel structure fail. The first gate in the fault tree is an OR gate and is shown in Figure 3.2.



MINIMUM CUT

1

G1

Figure 3.2. First Level of the Fault Tree

The minimum cut representation is listed beside the fault tree. Because the gate was an OR gate the inputs were listed as separate rows of a list matrix. The next level is an AND

gate and its inputs are two more gates as seen in Figure 3.3. The AND gate causes the inputs to be listed in the same row of the list matrix. The complete fault tree and its corresponding minimum cuts are given in Figure 3.4.

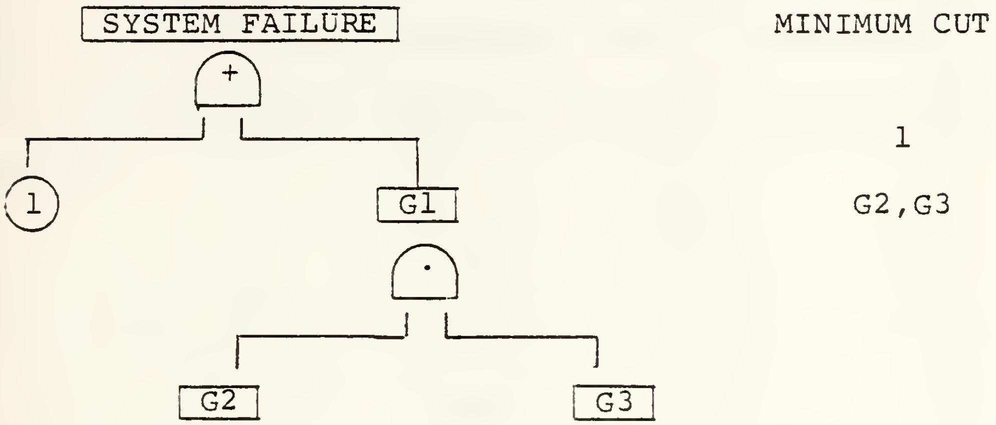


Figure 3.3. Second Level of the Fault Tree

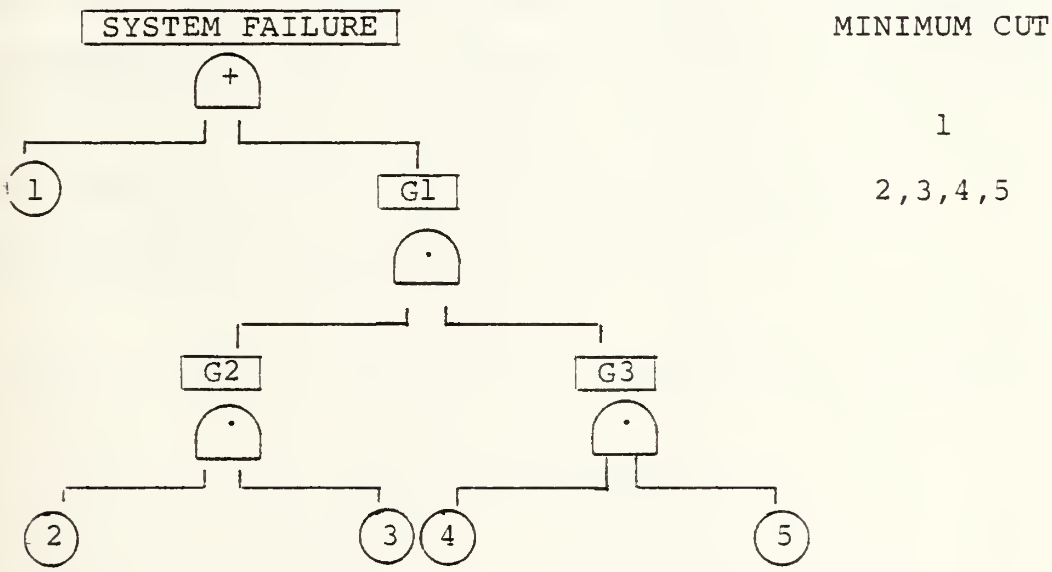


Figure 3.4. The Complete Fault Tree and Its Minimum Cut Representation

For a system such as that of Example 3a an algorithm can also be developed to find the resulting exponential survival function once all minimum cuts have been found. Step one is to find the survival function for each row. This step is relatively easy since the union of the events in each row will yield the proper exponents for the exponential functions. For the second row of the list matrix in Example 3a this would be:

$$\begin{aligned} 2,3,4,5 = {} & (2) + (3) + (4) + (5) - (2+3) - (2+4) - (2+5) \\ & - (3+4) - (3+5) - (4+5) + (2+3+4) + (2+3+5) \\ & + (3+4+5) + (2+4+5) - (2+3+4+5) \end{aligned}$$

Now insert the respective failure rates and the quantities in parenthesis times -t are the exponents for the exponentials in the survival function. The sign in front of the parenthesis is also the sign of the exponential. The survival function for the second row would be:

$$\begin{aligned} & e^{-\lambda_2 t} + e^{-\lambda_3 t} + e^{-\lambda_4 t} + e^{-\lambda_5 t} + e^{-(\lambda_2+\lambda_3)t} + e^{-(\lambda_2+\lambda_4)t} \\ & \quad + \ldots + e^{(-\lambda_2+\lambda_3+\lambda_4+\lambda_5)t} \end{aligned}$$

Step two is to take the product of the exponential survival functions of the rows. For the above example:

$$(e^{-\lambda_1 t})(e^{-\lambda_2 t}+e^{-\lambda_3 t}+e^{-\lambda_4 t}+e^{-\lambda_5 t}+e^{-(\lambda_2+\lambda_3)t}+\ldots+e^{-(\lambda_2+\lambda_3+\lambda_4+\lambda_5)t})$$

$$= e^{-(\lambda_1+\lambda_2)t}+e^{-(\lambda_1+\lambda_3)t}+e^{-(\lambda_1+\lambda_4)t} + \ldots + e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}$$

This expression represents the complete survival function for the system in Example 3a.

There exists an isomorphism between the paths of the branching diagram and all possible sequences of failures of components in the fault tree. The bottom level of the fault tree corresponds with the base of the branching diagram, all components are functioning at the start of the system. The failure of any component or components will cause branching in the branching diagram and gates to be affected in the fault tree. There is exactly the same number of initial branches as components in the fault tree. If the failure of a component caused system failure in the fault tree its respective branch in the branching diagram will end in a ZERO distribution. If the component failure did not cause system failure then the distribution of time to next failure among the remaining components will be at the end of the branch respective to the component that failed. Once again there is a one to one correspondence between components remaining in the fault tree and branches of the branching diagram at that point. This process will account for all possible permutations of events leading to system failure.

The branching diagram for the system in Example 3a is given in Figure 3.5. Due to the size of the branching diagram, only a few of the branches of the diagram are drawn out to completion. If drawn out completely there would be 65 possible paths through the branching diagram. This is the same as the number of possible sequences of failures in the fault tree. Not coincidently this is the same number of possible permutations and combinations of the two minimum cuts summed together.

For a coherent system with n components there are n! possible paths through the branching diagram if the only minimum cut set is the set of all components. If the minimum cut sets are proper subsets of the set of all components a path of the branching diagram will end in a ZERO distribution once the set of failed components corresponding to that path contains a minimum cut set. That is, a system will remain up and paths will continue to branch on component failure until the set of components along a path contains one of the minimum cut sets found by the fault tree algorithm. The probability of the occurrence of the minimum cut set or any particular sequence of failures is the product of the a priori probabilities of the corresponding path in the branching diagram.

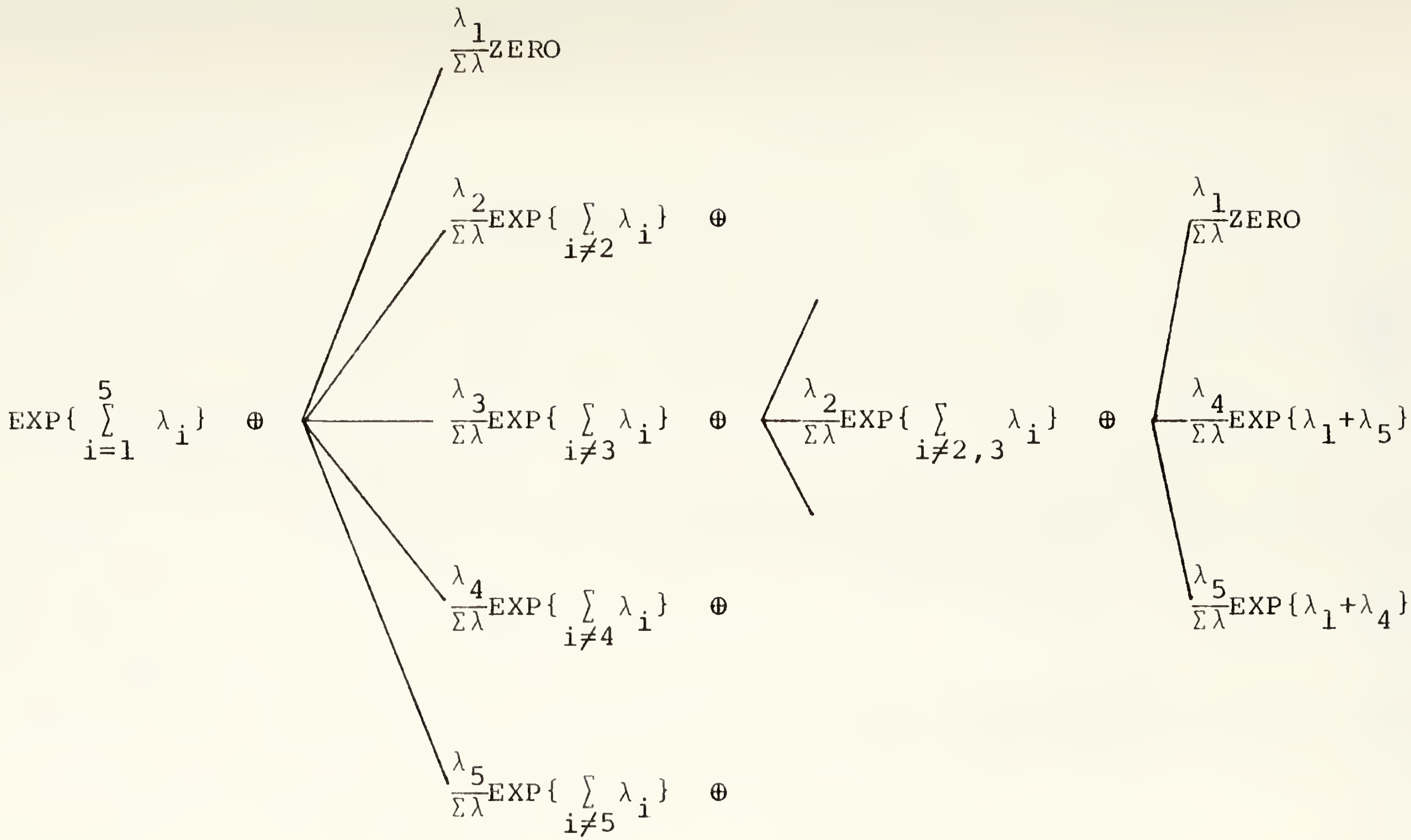


Figure 3.5. Branching Diagram for Example 3a

LIST OF REFERENCES

1. Barlow and Proschan, Statistical Theory of Reliability and Life Testing, P. 265-266, Holt, Rinehart, and Winston, 1975.

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